Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Linear Algebra -II **Subject Code : 4SC04MTC2**

Branch : B. Sc. (Mathematics,Physics) **Semester : IV**
Date : 20 /11/ 2015 **Time : 2:30 PM To 5:30 PM** **Marks : 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) State Cayley-Hamilton theorem.
b) Define: Orthogonal vector.
c) Express the quadratic form $2x^2 + 3y^2 - 5z^2 - 2xy + 6xz - 10yz$ in matrix notation.
d) Is the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$ negative semi-definite? Justify your answer.
e) Is the matrix $\begin{bmatrix} -100 & 100 \\ 0 & 100 \end{bmatrix}$ similar to diagonal matrix? Justify your answer.
f) If $T: V \to V$ has a nonzero kernel than what is the value of $\det(T)$.
g) Check whether the set $\left\{\frac{(e_1+e_2)}{\sqrt{2}}, \frac{(e_1-e_2)}{\sqrt{2}}\right\}$ orthonormal set.
h) Define: Eigenvalue of matrix.
i) If A is an $n \times n$ matrix then find the value of $\alpha_{11}C_{12} + \alpha_{21}C_{22} + \cdots + \alpha_{n1}C_{n2}$.
j) Identify the quadrics of equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$.
k) Define: Symmetric map.
l) Define: Orthogonal transformation.
m) Identify the conic of equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$.
n) Evaluate: $\det(e_1, e_2, \ldots, e_n)$.

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**
a) Reduce $3x^2 + 2xy + 4yz + 2xz - 2x - 14y - 2z - 9 = 0$ into standard form.
b) Show that the set $S = \{\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right), \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right), \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)\}$forms an orthonormal set.

[P.T.O]

Q-3 **Attempt all questions** **(14)**

a) Prove that the sum of the squares of the diagonals of a parallelogram equals the sum of the square of its sides.

b) Let $A, X \in M(n.R)$. Let A be invertible. Then we have

(1) $\det(A^{-1}) = (\det A)^{-1}$,

(2)$\det(A\,X\,A^{-1}) = \det(X)$.

c) Evaluate: $\det\begin{pmatrix} 1 & 5 & 0 & 0 \\ 2 & 0 & 8 & 0 \\ 3 & 6 & 9 & 0 \\ 4 & 7 & 8 & 1 \end{pmatrix}$.

Q-4 **Attempt all questions** **(14)**

a) Determine diagonal matrix orthogonally similar to the real symmetric matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. Also find modal matrix.

b) State and prove Riesz representation theorem.

Q-5 **Attempt all questions** **(14)**

a) Solve the following system of equations:

$x + y = 0, y + z = 1, z + x = -1$ using Cramer's rule.

b) Find the eigenvalues and eigenvectors for the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.

Q-6 **Attempt all questions** **(14)**

a) If A^t denotes the transpose of the matrix A, then show that $\det A = \det A^t$.

b) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

Q-7 **Attempt all questions** **(14)**

a) Use the Gram-Schmidt process to transform the basis vector $u_1 = (1,0,0), u_2 = (3,7,-2), u_3 = (0,4,1)$ into an orthonormal basis.

b) Find the inverse of the matrix $A = \begin{bmatrix} 0 & -1 & 3 \\ 2 & 5 & -4 \\ -3 & 7 & 1 \end{bmatrix}$.

Q-8 **Attempt all questions** **(14)**

a) Let $T: V \to V$ be linear map. Show that T is orthogonal if and only if $\|Tx\| = \|x\|$ for all $x \in V$.

b) For an $n \times n$ matrix $A = (\alpha_{ij})$, show that

$\det(A) = \alpha_{i1}C_{i1} + \alpha_{i2}C_{i2} + \cdots + \alpha_{in}C_{in}$, where C_{ij} is the cofactor of α_{ij}.